STRING MODELS OF GLUEBALL AND REGGE TRAJECTORIES *

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Abstract

The closed relativistic string carrying two point-like masses is considered as the model of a glueball with two constituent gluons. Here the gluon-gluon interaction is simulated by a pair of strings. For this system exact solutions of classical equations of motions are obtained. They describe rotational states of the string resulting in the set of quasilinear Regge trajectories with different behavior.

Introduction

In various string models of mesons and baryons [1] – [8] the Nambu-Goto string simulates strong interaction between quarks at large distances and the QCD confinement mechanism. This string has linearly growing energy (energy density is equal to the string tension γ) and accounts for the nonperturbative contribution of the gluon field.

All cited string hadron models generate linear or quasilinear Regge trajectories

$$J \simeq \alpha_0 + \alpha' M^2, \tag{1}$$

where J and M are the angular momentum and energy of a hadron state, the slope $\alpha' \simeq 0.9$ GeV^{-2} for the simplest model of meson is $\alpha' = 1/(2\pi\gamma)$. The string models describe excited hadron states on the leading Regge trajectories if we use the rotational states of these systems (planar uniform rotations) for this purpose.

These properties of the string were used for describing glueballs (bound states of gluons) on the base of different string models [9] – [12], including combinations of string and potential approaches [13] – [16]. The glueballs are predicted in QCD and observed in lattice QCD simulations [17, 18, 19].

The QCD glueball may be identified with the pomeron that is the Regge pole determining an asymptotic behavior of high-energy diffractive amplitudes [20, 21]. The pomeron Regge trajectory [19, 20]

$$J \simeq 1.08 + 0.25M^2 \tag{2}$$

differs from hadronic ones (1), in particular, its slope is essentially lower.

String models of glueballs describing trajectories of the type (2) or some exotic hadron states (glueball candidates) were suggested in the following variants [9] – [15]: (a) the open

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string with enhanced tension (the adjoint string) and two constintuent gluons at the endpoints; (b) the closed string simulating gluonic field; (c) the closed string carrying two point-like masses (constituent gluons). Evidently, the last model may be easily generalized for three-gluon glueballs [16].

In this paper the string model (c) (closed string with two masses m_1 and m_2) is considered, but it includes as particular cases the model (a) (if $m_1 = m_2 = 0$) and the closed string carrying one massive point ($m_2 = 0$) [22]. The last model may describe gluelumps [14], [23].

In Sect. 1 we describe the classical dynamics of the considered string model (c) moving in Minkowski space $R^{1,3}$ and in its generalization — in the space $\mathcal{M} = R^{1,3} \times T^{D-4}$ [22]. Here $T^{D-4} = S^1 \times S^1 \times \ldots \times S^1$ is $D-4$-dimensional torus with flat metric resulting from the compactification procedure of the string theory [24]. The space $\mathcal{M}$ has nontrivial structure of its homotopic classes. It is essential for states of a closed string. In the particular case $D = 4$ we simply have $\mathcal{M} = R^{1,3}$, so this case is also included into consideration.

In Sect. 2 rotational motions (planar uniform rotations) of this system are described and classified. They have much more complicated structure than a well known set of rotations of the folded rectilinear string. Rotational motions of string systems are widely used for generating Regge trajectories. Their structure and behavior for the considered system are described in Sect. 3.

1. Dynamics

The dynamics of the closed string carrying two point-like masses m_1 and m_2 is determined by the action close to that of the string baryon model "triangle" [5]:

$$S = -\gamma \int\limits_{\Omega} \sqrt{-g}\, d\tau d\sigma - \sum_{i=1}^{2} m_i \int \sqrt{\dot{x}_i^2(\tau)}\, d\tau. \tag{3}$$

Here γ is the string tension, g is the determinant of the induced metric $g_{ab} = \eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$ on the string world surface $X^\mu(\tau,\sigma)$, embedded in $\mathcal{M} = R^{1,3} \times T^{D-4}$ (including the particular case $D = 4$, $\mathcal{M} = R^{1,3}$), the speed of light $c = 1$. The world surface mapping in $\mathcal{M}$ from $\Omega = \{\tau, \sigma : \tau_1 < \tau < \tau_2,\ \sigma_0(\tau) < \sigma < \sigma_2(\tau)\}$ is divided by the world lines of massive points $x_i^\mu(\tau) = X^\mu(\tau, \sigma_i(\tau))$, $i = 0, 1, 2$ into two world sheets. Two of these functions x_0 and x_2 describe the same trajectory of the 2-nd massive point, and their equality forms the closure condition

$$X^\mu(\tau, \sigma_1(\tau)) \stackrel{\sim}{=} X^\mu(\tau^*, \sigma_2(\tau^*)) \tag{4}$$

on the tube-like world surface [5, 22]. These equations may contain two different parameters τ and τ^*, connected via the relation $\tau^* = \tau^*(\tau)$. This relation should be included in the closure condition (4) of the world surface.

The symbol of equality $\stackrel{\sim}{=}$ in the condition (4) means usual equality for coordinates x^0, x^1, x^2, x^3 in $R^{1,3}$, but for other coordinates x^k, $k = 4, 5\ldots$ (describing the torus T^{D-4}) this includes their cyclicity. Namely, points marked by x^k and $x^k + N_k\ell_k$, $N_k \in Z$ are identified: $x^k \stackrel{\sim}{=} x^k + N\ell_k$.

Equations of motion of this system result from the action (3) and its variation. They may be reduced to the simplest form under the orthonormality conditions on the world surface

$$(\partial_\tau X \pm \partial_\sigma X)^2 = 0, \tag{5}$$

and the conditions

$$\sigma_0(\tau) = 0, \qquad \sigma_2(\tau) = 2\pi. \tag{6}$$

Conditions (5), (6) always may be fixed without loss of generality, if we choose the relevant coordinates τ, σ [5]. It is connected with the invariance of the action (3) with respect to nondegenerate reparametrizations on the world surface $\tau = \tau(\tilde{\tau},\tilde{\sigma})$, $\sigma = \sigma(\tilde{\tau},\tilde{\sigma})$. The scalar square in Eq. (5) results from scalar product $(\xi,\zeta) = \eta_{\mu\nu}\xi^\mu\zeta^\nu$.

The orthonormality conditions (5) are equivalent to the conformal flatness of the induced metric g_{ab}. Under conditions (5), (6) the string motion equations take the form [5, 6, 22]

$$\frac{\partial^2 X^\mu}{\partial\tau^2} - \frac{\partial^2 X^\mu}{\partial\sigma^2} = 0, \tag{7}$$

$$m_1\frac{d}{d\tau}\frac{\dot{x}_1^\mu(\tau)}{\sqrt{\dot{x}_1^2(\tau)}} + \gamma\Big[X'^\mu + \dot{\sigma}_1(\tau)\dot{X}^\mu\Big]\Big|_{\sigma=\sigma_1-0} - \gamma\Big[X'^\mu + \dot{\sigma}_1(\tau)\dot{X}^\mu\Big]\Big|_{\sigma=\sigma_1+0} = 0. \tag{8}$$

$$m_2\frac{d}{d\tau}\frac{\dot{x}_2^\mu(\tau)}{\sqrt{\dot{x}_2^2(\tau)}} + \gamma\Big[X'^\mu(\tau^*(\tau),2\pi) - X'^\mu(\tau,0)\Big] = 0. \tag{9}$$

Here $\dot{X}^\mu \equiv \partial_\tau X^\mu$, $X'^\mu \equiv \partial_\sigma X^\mu$.

Eqs. (8), (9) are equations of motion for the massive points resulting from the action (3). They may be interpreted as boundary conditions for Eq. (7).

We denote the unit vectors

$$e_0^\mu,\ e_1^\mu,\ e_2^\mu,\ \dots\ e_{D-1}^\mu,$$

associated with coordinates x^μ. These vectors form the orthonormal basis in the manifold $\mathcal{M}$.

The closure condition (4) under equality (6) and the mentioned cyclicity of coordinates x^k, $k > 3$ takes the form

$$X^\mu(\tau^*,2\pi) = X^\mu(\tau,0) + \sum_{k\ge 4} N_k\ell_k e_k^\mu \tag{10}$$

The system of equations $(5)-10)$ describe dynamics of the closed string carrying two point-like masses without loss of generality. One also should add that the tube-like world surface of the closed string is continuous one, but its derivatives may have discontinuities at the world lines of the massive points (except for derivatives along these lines) [5]. These discontinuities are taken into account in Eq. (8).

2. Rotational states

We search rotational solutions of system $(5)-10)$ using the approach supposed in Ref. [5] for the string model "triangle" and in Ref. [22] for the closed string carrying one massive point. In the frameworks of the orthonormality gauge (5) we suppose that the system uniformly rotates, masses move at constant speeds v_1 and v_2 along circles and conditions

$$\sigma_1(\tau) = \sigma_1 = \text{const}, \tag{11}$$

$$\tau^* = \tau + \tau_0, \qquad \tau_0 \equiv 2\pi\theta = \text{const}, \tag{12}$$

and $\dot{X}^2(\tau,\sigma_i) = \text{const}$ or

$$\frac{\gamma}{m_1}\sqrt{\dot{X}^2(\tau,\sigma_1)} = Q_1, \qquad \frac{\gamma}{m_2}\sqrt{\dot{X}^2(\tau,0)} = Q_2, \qquad Q_i = \text{const}, \tag{13}$$

are fulfilled.

When we search solution of the linearized system (5) – (10) under restrictions (11) – (13) as a linear combination of terms $X^\mu(\tau,\sigma) = T^\mu(\tau)\, u(\sigma)$ (Fourier method) we obtain from Eq. (7) two equations for functions $T^\mu(\tau)$ and $u(\sigma)$:

$$T''_\mu(\tau) + \omega^2 T_\mu = 0, \qquad u''(\sigma) + \omega^2 u = 0.$$

Their solutions describing uniform rotations of the string system (rotational states) contain one nonzero frequency ω and have the following form [5, 22]:

$$X^\mu(\tau,\sigma) = x_0^\mu + e_0^\mu(a_0\tau + b_0\sigma) + \sum_{k\ge 3} e_k^\mu(a_k\tau + b_k\sigma) + u(\sigma)\cdot e^\mu(\omega\tau) + \tilde u(\sigma)\cdot \acute{e}^\mu(\omega\tau). \qquad (14)$$

Here

$$e^\mu(\omega\tau) = e_1^\mu \cos\omega\tau + e_2^\mu \sin\omega\tau, \qquad \acute{e}^\mu(\omega\tau) = -e_1^\mu \sin\omega\tau + e_2^\mu \cos\omega\tau$$

are unit orthogonal vectors rotating in the plane e_1, e_2; the functions

$$u(\sigma) = \left\{ \begin{array}{ll} A_1\cos\omega\sigma + B_1\sin\omega\sigma, & \sigma\in[0,\sigma_1],\\ A_2\cos\omega\sigma + B_2\sin\omega\sigma, & \sigma\in[\sigma_1,2\pi], \end{array}\right. \quad \tilde u(\sigma) = \left\{ \begin{array}{ll} \tilde A_1\cos\omega\sigma + \tilde B_1\sin\omega\sigma, & \sigma\in[0,\sigma_1],\\ \tilde A_2\cos\omega\sigma + \tilde B_2\sin\omega\sigma, & \sigma\in[\sigma_1,2\pi], \end{array}\right.$$

are continuous, but their derivatives have discontinuities at $\sigma = \sigma_1$ (the position of mass m_1).

Continuity of functions $u(\sigma)$ and $\tilde u(\sigma)$ at $\sigma = \sigma_1$ results in equalities

$$A_1C_1 + B_1S_1 = A_2C_1 + B_2S_1, \qquad \tilde A_1C_1 + \tilde B_1S_1 = \tilde A_2C_1 + \tilde B_2S_1. \qquad (15)$$

Here and below we use the notations

$$\begin{array}{llll} C_1 = \cos\omega\sigma_1, & C = \cos 2\pi\omega, & C_2 = \cos(2\pi-\sigma_1)\omega, & C_\theta = \cos 2\pi\theta\omega,\\ S_1 = \sin\omega\sigma_1, & S = \sin 2\pi\omega, & S_2 = \sin(2\pi-\sigma_1)\omega, & S_\theta = \sin 2\pi\theta\omega. \end{array} \qquad (16)$$

Expression (14) is the solution of Eq. (7) and it must satisfy the conditions (5), (8), (9), (10) under restrictions (11) – (13). Boundary condition (8) with adding Eq. (13) takes the form

$$\ddot X(\tau,\sigma_1) + Q_1\Big[X'^\mu(\tau,\sigma_1-0) - X'^\mu(\tau,\sigma_1+0)\Big] = 0.$$

Substituting Eq. (14) into this relation we obtain the equation

$$A_2S_1 - B_2C_1 = A_1(S_1 + h_1C_1) + B_1(h_1S_1 - C_1) \qquad (17)$$

and the same relation for $\tilde A_i$, $\tilde B_i$. Here we denote the constants

$$h_1 = \frac{\omega}{Q_1}, \qquad h_2 = \frac{\omega}{Q_2}. \qquad (18)$$

From the system (15), (17) one can express the amplitudes A_2, B_2 via A_1, B_1. The expressions for $\tilde A_2$, $\tilde B_2$ are the same, so it is convenient to use the matrix notations

$$\mathcal A = \begin{pmatrix} A_1\\ \tilde A_1\end{pmatrix}, \qquad \mathcal B = \begin{pmatrix} B_1\\ \tilde B_1\end{pmatrix},$$

and present these expressions in the form

$$\begin{pmatrix} A_2\\ \tilde A_2\end{pmatrix} = (1 + h_1C_1S_1)\,\mathcal A + h_1S_1^2\mathcal B, \qquad \begin{pmatrix} B_2\\ \tilde B_2\end{pmatrix} = -h_1C_1^2\mathcal A + (1 - h_1C_1S_1)\,\mathcal B. \qquad (19)$$

Substituting expression (14) into the closure condition (10) and taking into account Eqs. (19) we obtain the following relations for amplitudes:

$$b_0 = -\theta a_0, \qquad b_3 = -\theta a_3, \qquad b_k = -\theta a_k + \frac{\ell_k N_k}{2\pi} \quad (k > 3), \tag{20}$$

$$M_\theta \Big[(C - h_1 C_1 S_2)\, \mathcal{A} + (S - h_1 S_1 S_2)\, \mathcal{B} \Big] = \mathcal{A} \tag{21}$$

In the latter matrix equality we use the matrix

$$M_\theta = \begin{pmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{pmatrix}$$

and notations (16).

Boundary condition (9) with Eqs. (12), (13) for the solution (14) reduces into the matrix equation

$$M_\theta \Big[-(S + h_1 C_1 C_2)\, \mathcal{A} + (C - h_1 S_1 C_2)\, \mathcal{B} \Big] = h_2 \mathcal{A} + \mathcal{B}. \tag{22}$$

The system of matrix equations (21), (22) may be rewritten in the form

$$M_1 \mathcal{A} = M_2 \mathcal{B}, \qquad M_3 \mathcal{A} = M_4 \mathcal{B}, \tag{23}$$

where matrices

$$M_1 = (C - h_1 C_1 S_2)\, M_\theta - I, \quad M_2 = -(S - h_1 S_1 S_2)\, M_\theta,$$
$$M_3 = (S + h_1 C_1 C_2)\, M_\theta + h_2 I, \quad M_4 = (C - h_1 S_1 C_2)\, M_\theta - I$$

are linear combinations of M_θ and the identity matrix I.

Taking into account mutual commutativity of the matrices M_k and excluding the column $\mathcal{B}$ (or $\mathcal{A}$) from the system (23) we obtain the system equivalent to Eqs. (23)

$$M\mathcal{A} = 0, \qquad M\mathcal{B} = 0. \tag{24}$$

Here the matrix $M = M_1 M_4 - M_2 M_3$ may be reduced with using Eqs. (16) and equality $M_\theta^2 = 2C_\theta M_\theta - I$ to the following form:

$$M = \Big[2(C_\theta - C) + (h_1 + h_2)\, S - h_1 h_2 S_1 S_2 \Big] M_\theta.$$

The system (24) (or (23)) has nontrivial solutions if and only if $\det M = 0$ that is

$$2(C_\theta - C) + (h_1 + h_2)\, S - h_1 h_2 S_1 S_2 = 0. \tag{25}$$

This equation may be rewritten after expanding notations (16), (18) in the form

$$2(\cos 2\pi\theta\omega - \cos 2\pi\omega) + \frac{Q_1 + Q_2}{Q_1 Q_2} \omega \sin 2\pi\omega = \frac{\omega^2}{Q_1 Q_2} \sin \sigma_1 \omega \sin d_2 \omega,$$

where $d_2 = \sigma_2 - \sigma_1 = 2\pi - \sigma_1$. It connects unknown (for the present moment) values of parameters ω, θ, σ_1, Q_i.

Other relations connecting these parameters we obtain after substituting expression (14) into the orthonormality conditions (5):

$$\omega^2(A_i^2+B_i^2+\tilde{A}_i^2+\tilde{B}_i^2)=a_0^2(1+\theta^2)-\sum_{k\geq 3}(a_k^2+b_k^2),\qquad i=1,2; \tag{26}$$

$$\omega^2(\tilde{A}_iB_i-A_i\tilde{B}_i)=a_0^2\theta+\sum_{k\geq 3}a_kb_k,\qquad i=1,2. \tag{27}$$

Among two equations (27) only one is independent, for example, with $i=1$. If it's satisfied and the relations (19) take place — the second conditions (27) is satisfied too. But two equations (26) are independent. Below we use the first of them and their residual

$$C_1(h_1C_1+2S_1)(A_1^2+\tilde{A}_1^2)+S_1(h_1S_1-2C_1)(B_1^2+\tilde{B}_1^2)=2(C_1^2-S_1^2-h_1C_1S_1)(A_1B_1+\tilde{A}_1\tilde{B}_1). \tag{28}$$

Here Eqs. (19) are used.

Under condition (25) the matrix $M=0$ in Eq. (24) and an arbitrary nonzero column $\mathcal{A}$ or $\mathcal{B}$ is its eigenvector. It is connected with the rotational symmetry of the problem. So one can choose an optional pair $A_1\,\&\,\tilde{A}_1$, $B_1\,\&\,\tilde{B}_1$ or $A_1\,\&\,B_1$ and determine two other constants from Eqs. (23) (under condition (25) two matrix equations (23) are equivalent), in particular:

$$\tilde{A}_1=\frac{C_*A_1+S_*B_1}{S_\theta},\qquad \tilde{B}_1=-\frac{KA_1+C_*B_1}{S_\theta}. \tag{29}$$

Here the coefficients

$$C_*=C-h_1C_1S_2-C_\theta,\qquad S_*=S-h_1S_1S_2,\qquad K=S+h_1C_1C_2+h_2C-h_1h_2C_1S_2$$

are connected by the following relation resulting from Eq. (25):

$$C_*^2+S_\theta^2=KS_*. \tag{30}$$

Values (29) must obey Eqs. (26) – (28) descending from the orthonormality conditions (5). If we substitute relations (29) into the first two equations and use Eqs. (25), (30), we obtain correspondingly

$$\frac{\omega^2A_*^2}{S_\theta^2}\Big[2S+(h_1+h_2)\,C-h_1h_2C_1S_2\Big]=a_0^2(1+\theta^2)-\sum_{k\geq 3}(a_k^2+b_k^2), \tag{31}$$

$$\frac{\omega^2A_*^2}{S_\theta}=a_0^2\theta+\sum_{k\geq 3}a_kb_k. \tag{32}$$

Here the amplitude factor

$$A_*^2=KA_1^2+2C_*A_1B_1+S_*B_1^2. \tag{33}$$

If we substitute relations (29) into Eq. (28) it transforms into identity.

The solution (14) must obey the last restrictions (13). This fact and Eqs. (29), (30) result in relations

$$\begin{aligned} a_0^2-\sum_{k\geq 3}a_k^2-\omega^2A_*^2(S-h_2S_1S_2)\,S_\theta^{-2}&=m_1^2Q_1^2/\gamma^2,\\ a_0^2-\sum_{k\geq 3}a_k^2-\omega^2A_*^2(S-h_1S_1S_2)\,S_\theta^{-2}&=m_2^2Q_2^2/\gamma^2. \end{aligned} \tag{34}$$

Below we suppose

$$a_k=0\quad\text{for}\quad k\geq 3 \tag{35}$$

without loss of generality because the terms with a_k in Eq. (14) describe a uniform rectilinear motion of the system at a constant velocity. It may be eliminated via Lorentz transformation [22]. In the case $a_k = 0$, in particular, relations (20) transform into

$$b_0 = -\theta a_0, \qquad b_3 = 0, \qquad b_k = \frac{\ell_k N_k}{2\pi} \quad (k > 3), \tag{36}$$

and other equations (26) – (34) — correspondingly.

In the case (35) one can exclude the amplitude factor (33) from Eqs. (31), (32) and obtain the equation

$$1 + \theta^2 - \theta \frac{2S + (h_1 + h_2)\, C - h_1 h_2 C_1 S_2}{S_\theta} = \frac{1}{a_0^2} \sum_{k>3} b_k^2. \tag{37}$$

Relations between the factor a_0, speeds of the massive points $v_i = \mathrm{const}$ and other parameters of the system are result from Eqs. (31) – (34):

$$a_0 = \frac{m_1 Q_1}{\gamma\sqrt{1 - v_1^2}} = \frac{m_2 Q_2}{\gamma\sqrt{1 - v_2^2}}, \qquad A_*^2 = \frac{a_0^2 \theta S_\theta}{\omega^2}, \tag{38}$$

$$v_1^2 = \theta \frac{S - h_2 S_1 S_2}{S_\theta}, \qquad v_2^2 = \theta \frac{S - h_1 S_1 S_2}{S_\theta}. \tag{39}$$

Values ω and θ are determined from the system (25), and (37). Solution of the system (25), (37) (pairs ω, θ) form some countable set. Each pair corresponds to solution (14) describing uniform rotation of the closed string with certain topological type.

To investigate the obtained world surface (14) one can consider its section $t = t_0 = \mathrm{const}$ — a "photograph" of the string position at time moment t_0.

In the case $a_k = b_k = 0$ (or for Minkowski space $\mathcal{M} = R^{1,3}$) projections of these sections onto e_1, e_2 plane are closed curves, composed from segments of a hypocycloid if and only if the equalities (25), (37) are fulfilled. This result is similar to the behavior of rotational motions for the string baryon model "triangle" [5].

Hypocycloid is the curve drawing by a point of a circle (with radius r) rolling inside another fixed circle with larger radius R. In the case of solutions (14) uniformly rotating hypocycloidal segments of the string are joined at non-zero angles in the massive points. The relation of the mentioned radii

$$\frac{r}{R} = \frac{1 - |\theta|}{2}$$

is irrational if $m_i \neq 0$ and $\theta \neq 0$.

This hypocycloidal string rotates in the e_1, e_2 plane at the angular velocity $\Omega = \omega/a_0$, the massive points move at the speeds (39) along the circles with radii v_i/Ω. There are also cusps (return points) of the hypocycloid moving at the speed of light.

The more general solutions (14) including the summands with $b_k \neq 0$ (for cyclical coordinates in $\mathcal{M}$) differ from mentioned hypocycloidal solutions: their sections $t = \mathrm{const}$ are spatial (not flat) curves, closed because of cyclical nature of coordinates x_k. Their projections onto the plane e_1, e_2 look like hypocycloids. But these world surfaces have no peculiarities of the metric $\dot{X}^2 = X'^2 = 0$.

In the case when the parameter in Eq. (12) equals zero ($\theta = 0$ or $\tau_0 = 0$) solutions (14) describe rotational motions of n times folded string. It has a form of rotating rectilinear segment if $b_k = 0$ and more complicated form in the case $b_k \neq 0$. These motions are divided into two classes, we shall name them sa follows: (a) "linear states" with masses m_1, m_2 moving at nonzero velocities v_1, v_2 at the ends of the rotating rectilinear segment and (b) "central states", if one mass (or both masses) is placed at the rotational center.

The rotational motions (14) in the case $\theta \neq 0$ we shall name "hypocycloidal states".

There are many topologically different types of linear, central and hypocycloidal states (14). They may be classified with the number of cusps and the type of intersections of the hypocycloid following Ref. [5]. Note that in the considered model (3) the string does not interact with itself in a point of intersection.

These topological configurations of the rotational states may be classified by investigation of the massless $m_i \to 0$ or ultrarelativistic $v_i \to 1$ limit for fixed N_k, ℓ_k, γ, a_0. Analysis of equations (25), (37) – (39) shows that in the limit $m_i \to 0$ the values Q_i tend to infinity, values 2ω and $2\theta\omega$ tend to following integer numbers:

$$n_1 = \Big|\lim_{m_i\to 0} 2\omega\Big|, \qquad n_2 = \lim_{m_i\to 0} 2\theta\omega. \tag{40}$$

Because of the inequality $|\theta| < 1$ and condition (25), resulting in the equality $(-1)^{n_1} = (-1)^{n_2}$ (n_1 and n_2 are admissible:

$$n_1 \geq 2; \qquad n_2 = n_1 - 2,\ \ n_1 - 4,\ \dots\ -(n_1 - 2). \tag{41}$$

The number n_1 is the number of cusps of the rotating hypocycloid (including massive points), the number n_2 describes the shape of this curve. For example, values $n_1 = 5$, $n_2 = 3$ correspond to a curvilinear pentagon, values $n_1 = 5$, $n_2 = 1$ — a curvilinear star).

The case $n_2 = 0$ (this means $\theta = 0$) includes two classes: linear and central rotational states. They are joined in the limit $m_i \to 0$.

3. Regge trajectories

The obtained rotational motions of the considered model should be applied for describing physical manifestations of glueballs, in particular, their Regge trajectories. For this purpose we calculate the energy E and angular momentum J for the states (14) of this model. For an arbitrary classic state of the relativistic string with the action (3) carrying pointlike masses they are determined by the following integrals (Noether currents) [5, 22]:

$$P^\mu = \int_c p^\mu(\tau,\sigma)\,d\sigma + \sum_{i=1}^{2} p_i^\mu(\tau), \tag{42}$$

$$\mathcal{J}^{\mu\nu} = \int_C \Big[X^\mu(\tau,\sigma)\,p^\nu(\tau,\sigma) - X^\nu(\tau,\sigma)\,p^\mu(\tau,\sigma)\Big]\,d\sigma + \sum_{i=1}^{2}\Big[x_i^\mu(\tau)\,p_i^\nu(\tau) - x_i^\nu(\tau)\,p_i^\mu(\tau)\Big], \tag{43}$$

where $p^\mu(\tau,\sigma) = \gamma[(\dot X, X')X'^\mu - X'^2\dot X^\mu]/\sqrt{-g}$ is the canonical string momentum, $x_i^\mu(\tau) = X^\mu(\tau,\sigma_i(\tau))$ and $p_i^\mu(\tau) = m_i\dot x_i^\mu(\tau)/\sqrt{\dot x_i^2(\tau)}$ are coordinates and momentum of the massive points, C is any closed curve (contour) on the tube-like world surface of the string. Note that the lines $\tau = \text{const}$ on the world surface (14) are not closed in the case $\tau_0 \neq 0$. So we can use the most suitable lines $\tau - \theta\sigma = \text{const}$ (that is $t = \text{const}$) as the contour C in integrals (42), (43).

The reparametrization $\tilde\tau = \tau - \theta\sigma$, $\tilde\sigma = \sigma - \theta\tau$ keeps the orthonormality conditions (5). Under them $p^\mu(\tau,\sigma) = \gamma\dot X^\mu(\tau,\sigma)$.

The square of energy E^2 equals the scalar square of the conserved vector of momentum (42): $P^2 = P_\mu P^\mu = E^2$. If we substitute the expression (14) in the case (35) $a_k = 0$ into Eq. (42) we'll obtain the following formula for the momentum:

$$P^\mu = \gamma a_0 \Big[2\pi(1-\theta^2) + \frac{1}{Q_1} + \frac{1}{Q_2}\Big]\, e_0^\mu + \gamma\theta \sum_{k>3} \ell_k N_k e_k^\mu. \tag{44}$$

In the simplest case $N_k = 0$ (or for Minkowski space) this expression takes the form

$$P^\mu = E e_0^\mu, \qquad E = 2\pi\gamma a_0(1-\theta^2) + \frac{m_1}{\sqrt{1-v_1^2}} + \frac{m_2}{\sqrt{1-v_2^2}}. \tag{45}$$

The classical angular momentum (43) is not conserved value for the considered system because of anisotropy of the space $\mathcal{M}$. Only the components $\mathcal{J}^{\mu\nu}$ with $\mu,\nu = 0,1,2,3$ (relating to the space $R^{1,3}$) are conserved. Among them only z-component of the angular momentum is nonzero:

$$\begin{gathered}\mathcal{J}^{\mu\nu} = j_3^{\mu\nu} J,\\ J = \frac{\gamma}{2\omega}\bigg\{2\pi\Big[a_0^2(1-\theta^2) - \sum_{k\ge 3}\frac{\ell_k^2 N_k^2}{4\pi^2}\Big] + a_0^2\Big(\frac{v_1^2}{Q_1} + \frac{v_2^2}{Q_2}\Big)\bigg\}.\end{gathered} \tag{46}$$

Here $j_3^{\mu\nu} = e_1^\mu e_2^\nu - e_1^\nu e_2^\mu = e^\mu \acute{e}^\nu - e^\nu \acute{e}^\mu$.

For given cyclical numbers N_k the states (14) are determined by the parameters a_0, ω, θ. If the values m_i, γ and the topological type of the rotational state (14) are fixed we obtain the one-parameter set of motions with different values E and J. These states lay at quasilinear Regge trajectories.

The mentioned Regge trajectories are nonlinear for small E and tend to linear if $E\to\infty$. Their slope in this limit depends on the values N_k and the fixed topological type.

The ultrarelativistic limit $E\to\infty$ corresponds to $v_i \to 1-0$ (except for central states) and for values ω and θ — to the limits (40). Substituting into Eqs. (25), (37), (39), (44), (46) asymptotic relations with small values $\varepsilon_1 = \sqrt{1-v_1^2}$, $\varepsilon_2 = \sqrt{1-v_2^2}$, $2\omega = n_1 - \varepsilon_\omega$, $n_1\theta = n_2 - \varepsilon_\theta$, we obtain in the limit $J\to\infty$, $E\to\infty$ the following asymptotic relation between these values for fixed type (n_1, n_2, b_k) of the state:

$$J \simeq \alpha' E^2 + \alpha_1 E^{1/2} + \alpha_0, \qquad E\to\infty, \tag{47}$$

where

$$\begin{gathered}\alpha' = \frac{1}{2\pi\gamma}\,\frac{n_1}{n_1^2 - n_2^2},\\ \alpha_1 = -\frac{\sqrt{2}\,n_1(m_1^{3/2} + m_2^{3/2})}{3\sqrt{\pi}\gamma(n_1^2-n_2^2)^{3/4}}, \qquad \alpha_0 = -\frac{2\pi\gamma(n_1^2 - 2n_2^2)}{n_1(n_1^2-n_2^2)}\sum_{k>3} b_k^2.\end{gathered} \tag{48}$$

This dependence is close to linear one (1), but the slope α' (48) for this system differs from the Nambu value $\alpha' = 1/(2\pi\gamma)$ by the factor $\chi = n_1/(n_1^2 - n_2^2)$. The maximal slope with the factor $\chi = 1/2$ corresponds to $n_1 = 2$, $n_2 = 0$ that is to the linear state with two masses at the ends, connected two strings without singularities. For the "triangle" states $n_1 = 3$, $n_2 = 1$ this factor is $\chi = 3/8$ and these states (and also states with $n_1 = 4$, $n_2 = 2$, $\chi = 1/3$) are more suitable candidates for describing the glueball trajectory (2).

Conclusion

The obtained rotational states (14) of the closed string with two point-like masses are divided in a set of different topological classes, describing by the integer parameters (40). The states from different classes generate the wide spectrum of quasilinear Regge trajectories (44), (46) with different slopes (46) in the limit (47) of large energies. There are some classes of these states suitable for describing the pomeron (glueball) trajectory (2).

The considered model needs further development, in particular, quantization or quantum corrections. These corrections are to be significant for calculation of the intercept α_0.

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References

[1] Y. Nambu, Phys. Rev. **D10**, 4262 (1974).

[2] A. Chodos and C. B. Thorn, Nucl. Phys. **B72**, 509 (1974).

[3] X. Artru, Nucl. Phys. **B85**, 442 (1975).

[4] I. Yu. Kobzarev, B. V. Martemyanov, and M. G. Shchepkin, Sov. Phys. Usp. **35**, 257 (1992).

[5] G. S. Sharov, Phys. Rev. **D58**, 114009 (1998); hep-th/9808099.

[6] G. S. Sharov, Phys. Atom. Nucl. **62**, 1705 (1999).

[7] A. Inopin, G. S. Sharov, Phys. Rev. **D63**, 054023 (2001), hep-ph/9905499.

[8] L. D. Soloviev, Phys. Rev. **D58**, 035005 (1998); **D61**, 015009 (2000), hep-ph/9907486.

[9] K. Yamada, S. Ishida, J. Otokozawa, N. Honzawa. Prog. Theor. Phys. **97**, 813 (1977).

[10] L. D. Soloviev, Theor. Math. Phys. **126**, 203 (2001), hep-ph/0006010.

[11] S. V. Talalov, hep-ph/0101028.

[12] L. A. Pando Zayas, J. Sonnenschein, and D. Vaman, Nucl. Phys. **B682**, 3 (2004), hep-th/0311190.

[13] F. Brau, C. Semay, and B. Silvestre-Brac, Phys. Rev. **D70**, 014017 (2004), hep-ph/0412173.

[14] E. Abreu, P. Bicudo, and A. Rovisco, hep-ph/0508281.

[15] V. Mathieu, C. Semay, and F. Brau, Eur. Phys. J. **A27**, 225 (2006), hep-ph/0511210.

[16] V. Mathieu, C. Semay, and B. Silvestre-Brac, Phys. Rev. **D74**, 054002 (2006), hep-ph/0605205.

[17] C. J. Morningstar and M. J. Peardon, Phys. Rev. D **60**, 034509 (1999), hep-lat/9901004.

[18] D. Q. Liu and J. M. Wu, Mod. Phys. Lett. A **17**, 1419 (2002), hep-lat/0105019.

[19] H. B. Meyer and M. Teper, Phys. Lett. B **605**, 344 (2005).

[20] S. Donnachie, G. Dosch, O. Nachtmann and P. Landshoff, *Pomeron Physics and QCD*, Cambridge University Press, 2002.

[21] A. B. Kaidalov, hep-ph/0103011.

[22] A. E. Milovidov and G. S. Sharov, Theor. Math. Phys. **142**, 61 (2005); hep-th/0401070.

[23] G. Karl and J. Paton, Phys. Rev. D **60**, 034015 (1999), hep-ph/9904407.

[24] M. B. Green, J. H. Schwarz, E. Witten, *Superstring theory*, Cambridge University Press, Cambridge, 1987. V. 1, 2.